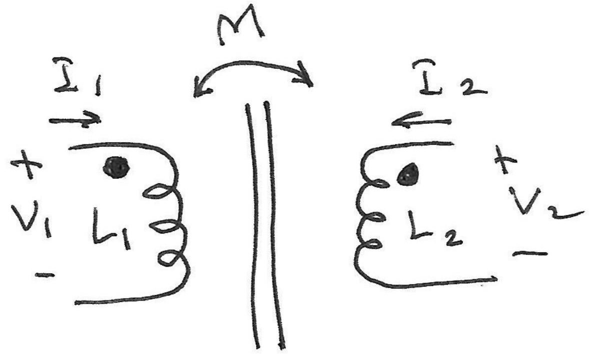
Mutual Inductance



Coefficient of coupling:

$$k = \sqrt{\frac{M}{L_1 L_2}}$$

If coupling is perfect:

$$k = 1$$

In the time domain:

$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

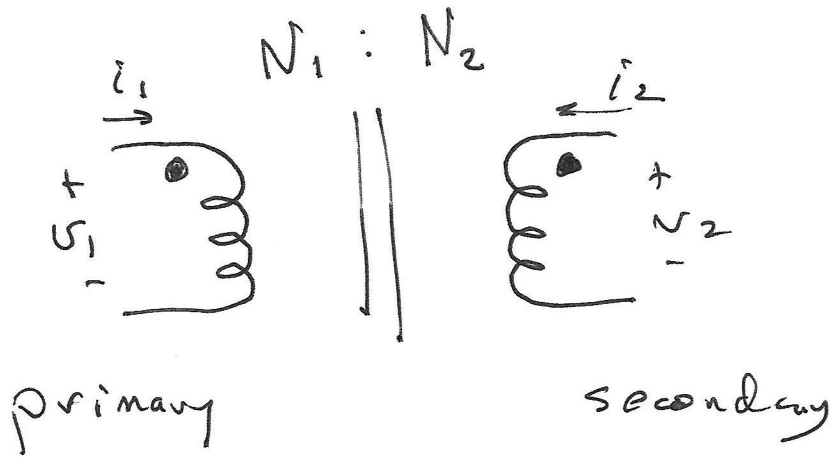
$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

In the phasor (frequency) domain:

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$

Real Transformer



alternate notation

$N_p : N_s$

primary secondary

$$v_2 = \frac{N_2}{N_1} v_1$$

$$i_2 = -\frac{N_1}{N_2} i_1$$

Faraday's Law

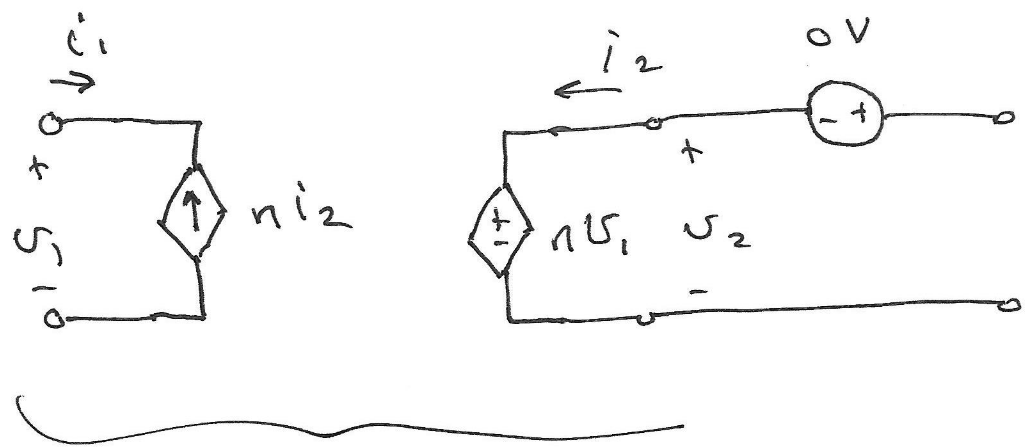
Lenz's Law

Define $n = \dfrac{N_2}{N_1}$ ← turns in secondary, ← turns in primary

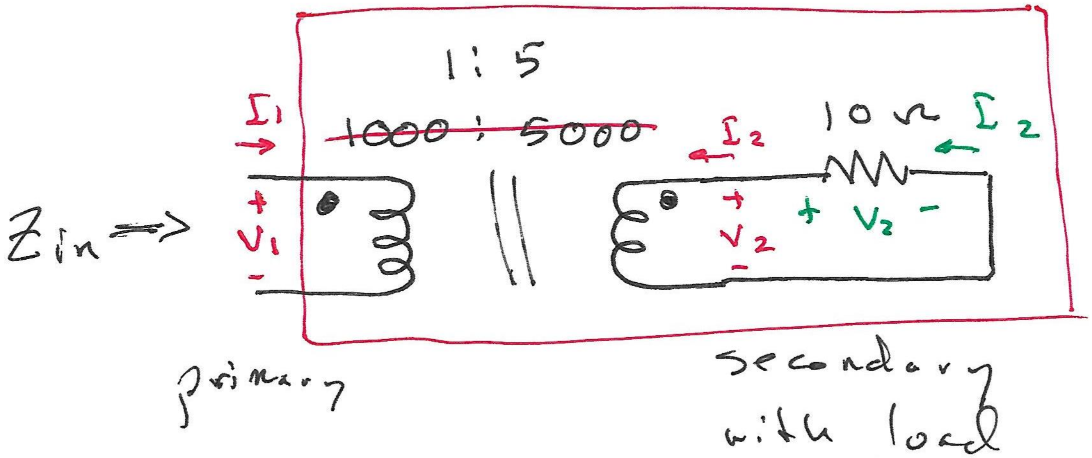
$$v_2 = n v_1$$

$$i_2 = -\frac{1}{n} i_1 \quad \Rightarrow \quad i_1 = -n i_2$$

$$i_1 v_1 = i_2 v_2$$



ideal transformer



$V_2 = -10 I_2$

$-\dfrac{V_2}{I_2} = 10$

$$\frac{N_2}{N_1} = \frac{5}{1} \quad \text{or} \quad n = 5$$

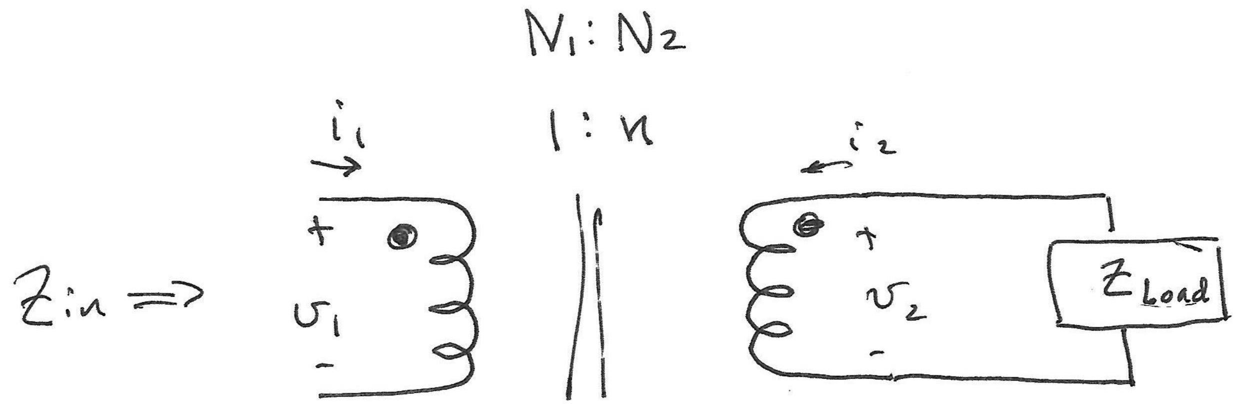
$$V_2 = 5 V_1$$

$$I_2 = -\frac{1}{5} I_1$$

$$Z_{in} \triangleq \frac{V_1}{I_1} = \frac{\frac{1}{5} V_2}{-5 I_2} = -\frac{1}{25} \frac{V_2}{I_2}$$

$$-\frac{V_2}{I_2} = 10\,\Omega$$

$$Z_{in} = -\frac{1}{25}(-10) = \frac{10}{25} = \frac{2}{5}\,\Omega$$



Determine Z_{in} in terms of n and Z_{Load}

$$v_2 = n\, v_1$$

$$i_2 = -\frac{1}{n}\, i_1$$

$$v_2 = -Z_{Load}\, i_2$$

$$V_2 = n\, V_1$$

$$I_2 = -\frac{1}{n}\, I_1$$

$$V_2 = -Z_{Load}\, I_2$$

$$Z_{in} = \frac{V_1}{I_1}$$

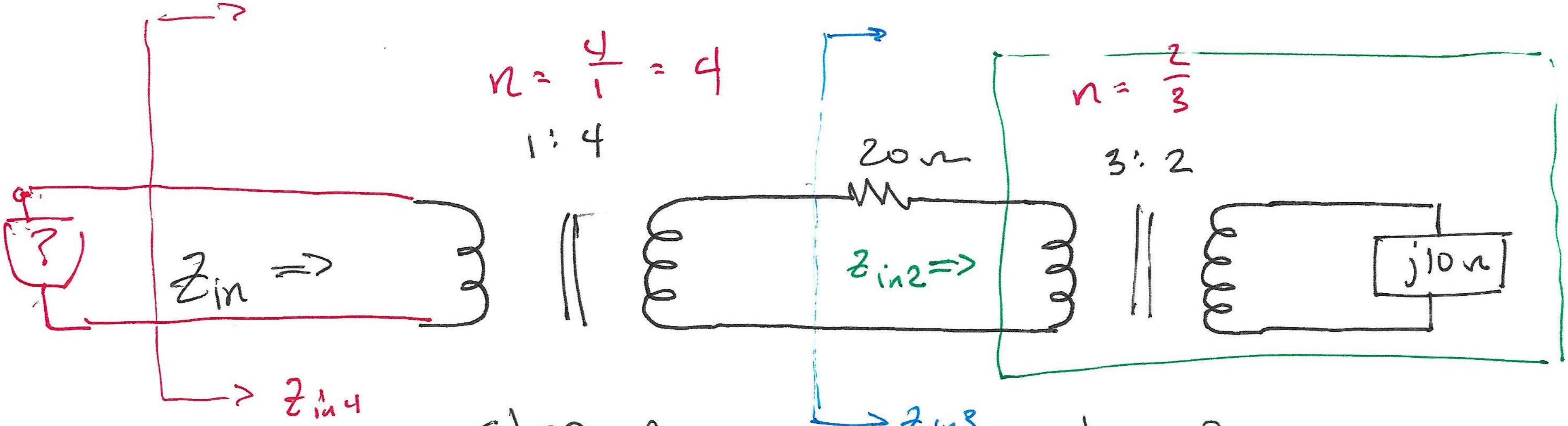
$$Z_{in} = \frac{V_1}{I_1}$$

$$= \frac{\frac{1}{n} V_2}{-n I_2}$$

$$= -\frac{1}{n^2} \frac{V_2}{I_2}$$

$$= -\frac{1}{n^2} \left(- Z_{Load} \right)$$

$$\boxed{Z_{in} = \frac{1}{n^2} Z_{Load}}$$



step-up transformer (higher voltage on secondary)

step-down transformer (lower voltage on secondary)

$$Z_{in2} = \frac{1}{\left(\frac{2}{3}\right)^2}(j10)$$

$$= j\frac{90}{4} \quad = j17.5\ \Omega$$

$$= j\frac{45}{2}\ \Omega$$

$$Z_{in3} = 20 + Z_{in2} \quad = 20 + j\frac{45}{2}\ \Omega$$

$$Z_{in} = \frac{1}{(4)^2} Z_{in3}$$

$$= \frac{1}{16}\left[20 + j\frac{45}{2}\right]$$

$$= \frac{5}{4} + j\frac{45}{32} \quad \Omega$$

Assume ω is given.

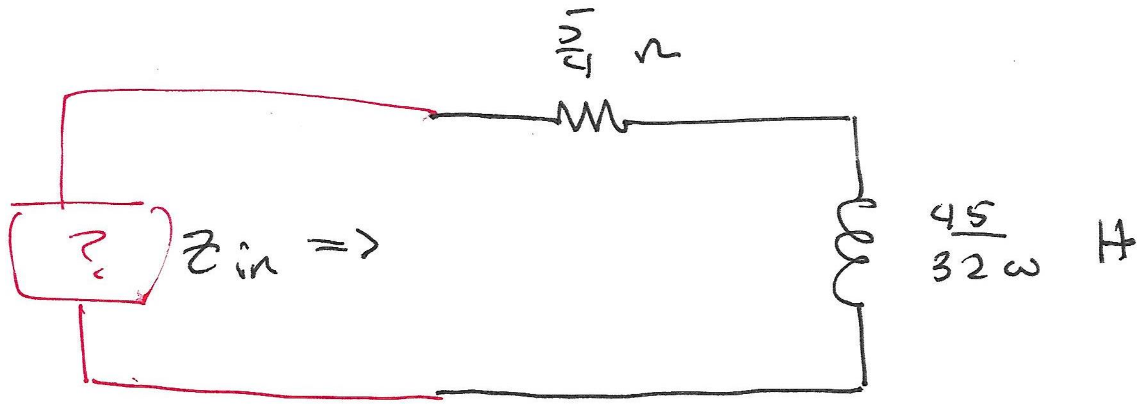
Z_{in} has two components:

$\frac{5}{4}$ real => resistor of $\frac{5}{4}\,\Omega$

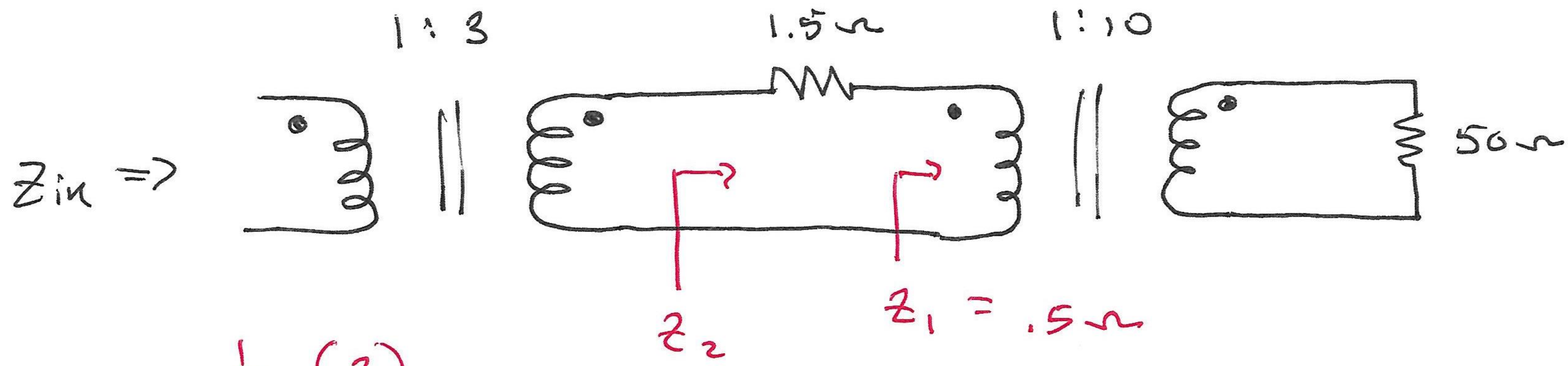
$\frac{45}{32}$ + imaginary => inductor with

$$j\omega L = j\frac{45}{32}$$

$$L = \frac{45}{32\omega}$$



5/4 Ω
?
Zin =>
45/32ω H



$\frac{1}{(3)^2}(2)$

$= \frac{2}{9}$

$\approx .22\ \Omega$